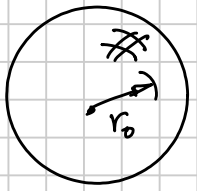


1.- A fused-quartz sphere has a thermal diffusivity of  $9.5 \times 10^{-7} \text{ m}^2/\text{s}$ , a diameter of 2.5 cm, and a thermal conductivity of  $1.52 \text{ W/m}^\circ\text{C}$ . The sphere is initially at a uniform temperature of  $25^\circ\text{C}$  and is suddenly subjected to a convection environment at  $200^\circ\text{C}$ . The convection heat-transfer coefficient is  $110 \text{ W/m}^2\cdot^\circ\text{C}$ . Calculate the temperatures at the center and at a radius of 6.4 mm after a time of 3 min.

## Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$D_o = 0.025 \text{ m}$   
 $r_o = 0.0125 \text{ m}$   
 $\alpha = 9.5 \times 10^{-7} \text{ m}^2/\text{s}$   
 $k = 1.52 \text{ W/m}^\circ\text{C}$   
 $T_i = 25^\circ\text{C}$   
 $T_\infty = 200^\circ\text{C}$   
 $h = 110 \text{ W/m}^2\cdot^\circ\text{C}$   
 $T|_{r=0} = ? \text{ @ } t = 3 \text{ min}$   
 $T|_{r=0.0064} = ? \text{ @ } t = 3 \text{ min}$

$$L_c = \frac{r_o}{3} = 0.004167$$

$$Bi = \frac{h L_c}{k} = \frac{110(0.004167)}{1.52} = 0.3 > 0.1$$

transient Heat conduction with spatial Effect

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(9.5 \times 10^{-7})(180)}{(0.0125)^2}$$

$$\tau = 1.0944 > 0.2$$

One term solution can be used

$$Bi = \frac{h r_o}{k}$$

$$Bi = \frac{(110)(0.0125)}{1.52}$$

$$Bi = 0.9046$$

$$\rightarrow \begin{aligned} A_1 &= 1.2499 \\ \lambda_1 &= 1.5075 \end{aligned}$$

## Knowledge (Equations that governing the phenomenon)

$$L_c = \frac{V}{A} \quad Bi = \frac{h L_c}{k}$$

$$\tau = \frac{\alpha t}{r_o^2}$$

$$Bi = \frac{h r_o}{k}$$

$$\theta_{sph} = \frac{T - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$

$$\theta_{o,sph} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

## Application (Solution procedure, all computations)

$$\frac{T_o - 200}{25 - 200} = (1.2499) e^{-(1.5075)^2 (1.0944)}$$

$$@ r = 0.0064$$

$$T_o = 200 - 175 (0.10393)$$

$$T_o = 181.8^\circ\text{C}$$

$$T|_{r=0.0064} = 200 - 175 (0.10393) \frac{\sin(1.5075 (\frac{0.0064}{0.0125}))}{1.5075 (\frac{0.0064}{0.0125})}$$

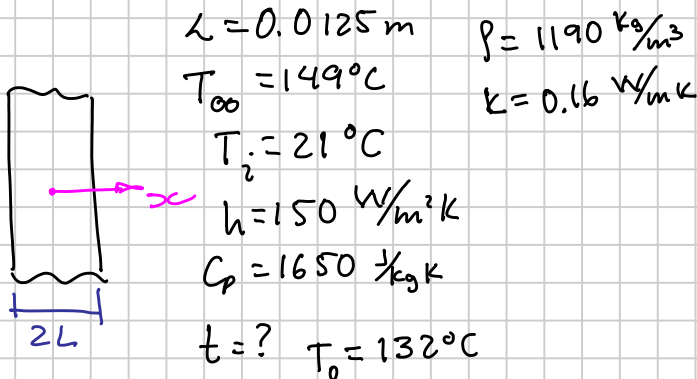
$$T|_{r=0.0064} = 200 - 175 (0.10393) (0.9036)$$

$$T|_{r=0.0064} = 183.6^\circ\text{C}$$

## Results and/or Analysis of Results

2.- In the vulcanization of tires, the carcass is placed into a jig, and steam at  $149^\circ\text{C}$  is admitted suddenly to both sides. If the tire thickness is 2.5 cm, the initial temperature is  $21^\circ\text{C}$ , the heat transfer coefficient between the tire and the steam is  $150\text{ W/(m}^2\text{ K)}$ , and the specific heat of the rubber is  $1650\text{ J/(kg K)}$ , estimate the time required for the center of the rubber to reach  $132^\circ\text{C}$ . (use hard vulcanized rubber properties)

Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



Assuming  $\tau > 0.2$   
for using one-term-solution

$$Bi = \frac{hL}{k} = \frac{(150)(0.0125)}{0.16}$$

$$Bi = 11.7188$$

$$\lambda_1 = 1.4405$$

$$A_1 = 1.2634$$

Knowledge (Equations that governing the phenomenon)

$$Bi = \frac{hL}{k} \quad \text{One-term}$$

for center

$$\theta_{0, \text{wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

Application (Solution procedure, all computations)

$$\alpha = \frac{k}{\rho C_p}$$

$$\alpha = \frac{0.16}{(1190)(1650)}$$

$$\alpha = 8.149 \times 10^{-8}$$

$$\frac{132 - 149}{21 - 149} = (1.2634) e^{-(1.4405)^2 \tau}$$

$$0.1328 = (1.2634) e^{-2.0750 \tau}$$

$$\ln(0.10511) = -2.0750 \tau$$

$$\tau = 1.0857$$

$$t = \frac{\tau L^2}{\alpha}$$

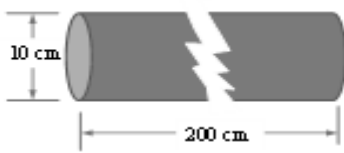
$$t = \frac{1.0857(0.0125)^2}{8.149 \times 10^{-8}}$$

$$t = 2081.74\text{ s}$$

$$t = 34.7\text{ min}$$

Results and/or Analysis of Results

3.- A stainless steel AISI 304 cylindrical billet is heated to 594°C preparatory to a forming process. If the minimum temperature permissible for forming is 460°C, how long may the billet be exposed to air at 38°C if the average heat transfer coefficient is 85 W/(m² K)? (evaluate properties at 300 K)



$$D_o = 0.1 \text{ m}$$

$$r_o = 0.05 \text{ m}$$

AISI-304  
@ T = 300 K

$$k = 14.9 \text{ W/mK}$$

$$C_p = 477 \text{ J/kgK}$$

$$\rho = 7900 \text{ kg/m}^3$$

$$T_\infty = 38^\circ\text{C}$$

$$h = 85 \text{ W/m}^2\text{K}$$

$$T_i = 594^\circ\text{C}$$

$$@ t = ? \quad T_s = 460^\circ\text{C}$$

Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)

$$\alpha = \frac{k}{\rho C_p} = \frac{14.9}{(7900)(477)} = 3.954 \times 10^{-6} \text{ m}^2/\text{s}$$

Assuming  $\tau > 0.2$

$$Bi = \frac{(85)(0.05)}{14.9}$$

$$Bi = 0.2852$$

$$\rightarrow \lambda_1 = 0.7273 \rightarrow J_0(\lambda_1) = 0.8717$$

$$A_1 = 1.0678$$

Knowledge (Equations that governing the phenomenon)

$$Bi = \frac{h r_o}{k} \quad \tau = \frac{\alpha t}{r_o^2}$$

one term solution

$$\theta_{cyl} = \frac{T - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o)$$

for surface  $r = r_o$

$$\frac{T_s - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1)$$

Application (Solution procedure, all computations)

$$\frac{460 - 38}{594 - 38} = (1.0678) e^{-(0.7273)^2 \tau} (0.8717)$$

$$0.7590 = 0.9308 e^{-0.5290 \tau}$$

$$\tau = \frac{\ln(0.7590/0.9308)}{-0.5290}$$

$$\tau = 0.3857 > 0.2 \quad \checkmark \text{ ok}$$

$$\tau = \frac{\alpha t}{r_o^2}$$

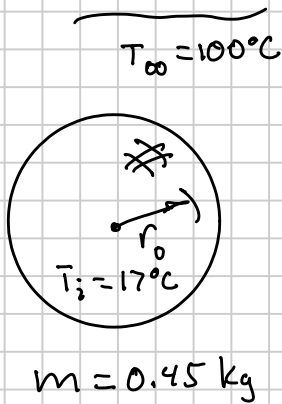
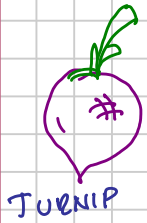
$$t = \frac{r_o^2 \tau}{\alpha} = \frac{(0.05)^2 (0.3857)}{3.954 \times 10^{-6}}$$

Results and/or Analysis of Results

$$t = 244 \text{ s}$$

$$t = 4 \text{ min}$$

4.- A monster turnip (assumed spherical) weighing in at 0.45 kg is dropped into a cauldron of water boiling at atmospheric pressure. If the initial temperature of the turnip is 17°C, how long does it take to reach 92°C at the center? Assume that  $h_c = 170 \text{ W/(m}^2 \text{ } ^\circ\text{C)}$   $c_p = 3900 \text{ J/(kg } ^\circ\text{C)}$   $k = 0.52 \text{ W/(m } ^\circ\text{C)}$   $\rho = 1040 \text{ kg/m}^3$ .



Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)

$$T_o = 92^\circ\text{C} @ t = ?$$

$$h_c = 170 \text{ W/m}^2 \text{ K}$$

$$C_p = 3900 \text{ J/kg } ^\circ\text{C}$$

$$k = 0.52 \text{ W/m } ^\circ\text{C}$$

$$\rho = 1040 \text{ kg/m}^3$$

Solving using  
One term Approximation  
 $\tau > 0.2$

$$V = \frac{0.45}{1040} = 4.327 \times 10^{-4} \text{ m}^3$$

$$r = \sqrt[3]{\frac{3}{4\pi} (4.327 \times 10^{-4})} = 0.04692 \text{ m}$$

$$Bi = \frac{(170)(0.04692)}{(0.52)}$$

$$Bi = 15.34$$

$$\lambda_1 = 2.9161$$

$$A_1 = 1.9533$$

Knowledge (Equations that governing the phenomenon)

$$\frac{T_o - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \quad Bi = \frac{h r_o}{k}$$

$$\alpha = \frac{k}{\rho c_p} \quad \tau = \frac{\alpha t}{L^2}$$

Application (Solution procedure, all computations)

$$\alpha = \frac{0.52}{(1040)(3900)} = 1.282 \times 10^{-7} \text{ m}^2/\text{s}$$

$$\frac{92 - 100}{17 - 100} = 1.9533 e^{-(2.9161)^2 \tau}$$

$$0.09639 = 1.9533 e^{-8.5036 \tau}$$

$$\tau = 0.3538 > 0.2$$

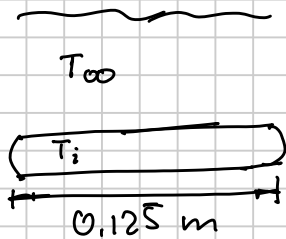
$$t = \frac{0.3538 (0.04692)^2}{(1.282 \times 10^{-7})}$$

$$t = 6076 \text{ s} \approx 1.69 \text{ hrs.}$$

Results and/or Analysis of Results

5.- A hot dog can be considered to be a 12.5-cm-long cylinder whose diameter is 2.5 cm and whose properties are  $\rho = 980 \text{ kg/m}^3$ ,  $c_p = 3.9 \text{ kJ/kg } ^\circ\text{C}$ ,  $k = 0.76 \text{ W/m } ^\circ\text{C}$ , and  $\alpha = 2 \times 10^{-7} \text{ m}^2/\text{s}$ . A hot dog initially at  $5^\circ\text{C}$  is dropped into boiling water at  $100^\circ\text{C}$ . The heat transfer coefficient at the surface of the hot dog is estimated to be  $600 \text{ W/m}^2 \cdot ^\circ\text{C}$ . If the hot dog is considered cooked when its center temperature reaches  $80^\circ\text{C}$ , determine how long it will take to cook it in the boiling water.

### Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$h = 600 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$r_o = 0.0125 \text{ m}$$

$$\rho = 980 \text{ kg/m}^3$$

$$k = 0.76 \text{ W/m } ^\circ\text{C}$$

$$\alpha = 2 \times 10^{-7} \text{ m}^2/\text{s}$$

$$T_i = 5^\circ\text{C}$$

$$T_\infty = 100^\circ\text{C}$$

$$t = ? \quad T_o = 80^\circ\text{C}$$

Assuming  $\tau > 0.2$

Using One term Approx.

$$Bi = \frac{(600)(0.0125)}{0.76} = 9.868$$

$$\lambda_1 = 2.1765$$

$$A_1 = 1.5668$$

### Knowledge (Equations that governing the phenomenon)

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

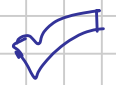
$$Bi = \frac{h r_o}{k} \quad \tau = \frac{\alpha t}{r_o^2}$$

$$\frac{80 - 100}{5 - 100} = 1.5668 e^{-2.1765^2 \tau}$$

$$\frac{0.2105}{1.5668} = e^{-4.737 \tau}$$

$$\tau = 0.4237$$

> 0.2



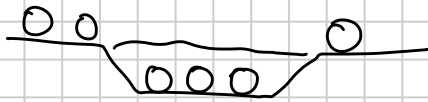
$$t = \frac{0.4237 (0.0125)^2}{2 \times 10^{-7}}$$

$$t = 331.0 \text{ s} \quad \approx 5.5 \text{ min}$$

### Application (Solution procedure, all computations)

### Results and/or Analysis of Results

6.- Ball bearings are to be hardened by quenching them in a water bath at a temperature of  $37^\circ\text{C}$ . Suppose you are asked to devise a continuous process in which the balls could roll from a soaking oven at a uniform temperature of  $870^\circ\text{C}$  into the water, where they are carried away by a rubber conveyer belt. The rubber conveyor belt would, however, not be satisfactory if the surface temperature of the balls leaving the water is above  $90^\circ\text{C}$ . If the surface coefficient of heat transfer between the balls and the water may be assumed to be equal to  $590 \text{ W}/(\text{m}^2 \text{ K})$ , calculate the cooling time, in seconds, required for a ball having a 6-cm diameter, and calculate the total amount of heat in watts which would have to be removed from the water bath in order to maintain its temperature uniform if 100,000 balls of 6-cm diameter are to be quenched per hour. Thermal conductivity  $k = 43 \text{ W}/(\text{m K})$ , Density  $\rho = 7801 \text{ kg}/\text{m}^3$ , Specific heat  $c = 473 \text{ J}/(\text{kg K})$ , Thermal diffusivity  $\alpha = 1.172 \times 10^{-5} \text{ m}^2/\text{s}$



Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)

$$r_o = 0.03 \text{ m}$$

$$L_c = \frac{r_o}{3} = 0.01$$

$$Bi = \frac{h L_c}{k} = \frac{(590)(0.01)}{43}$$

$$Bi = 0.1372 > 0.1$$

The Lumped Analysis can not be used

Assuming  $\tau > 0.2$  Applying One term Ap.

$$Bi = \frac{h r_o}{k} = \frac{(590)(0.03)}{43} = 0.4116$$

$$\lambda_1 = 1.0659$$

$$A_1 = 1.1196$$

for surface  $r = r_o$

$$\frac{T_s - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1)}{\lambda_1}$$

$$\frac{90 - 37}{870 - 37} = 1.1196 e^{-(1.0659)^2 \tau} \frac{\sin(1.0659)}{1.0659}$$

$$0.06363 = 0.9193 e^{-1.136 \tau}$$

$$\tau = 2.351$$

$$t = \frac{(2.351)(0.03)^2}{1.172 \times 10^{-5}} = 180.5 \text{ s}$$

Knowledge (Equations that governing the phenomenon)

$$Bi = \frac{h r_o}{k} \quad \tau = \frac{\alpha t}{r_o^2}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$

$$\frac{Q}{Q_{\max}} = 1 - 3 \theta_0 \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

$$Q_{\max} = m c_p (T_i - T_\infty)$$

Application (Solution procedure, all computations)

$$Q_{\max} = (7801) \frac{4\pi}{3} (0.03)^3 (473) (870 - 37)$$

$$Q_{\max} = 347623 \text{ J}$$

$$\theta_0 = A_1 e^{-\lambda_1^2 \tau}$$

$$\frac{Q}{Q_{\max}} = 1 - 3 (1.1196) e^{-(1.0659)^2 (2.351)} \frac{\sin(1.0659) - (1.0659) \cos(1.0659)}{(1.0659)^3}$$

$$= 1 - 3 (0.07745) (0.2920)$$

$$= 0.9310$$

$$Q = (0.931)(347.623) = 323.6 \text{ kJ/ball}$$

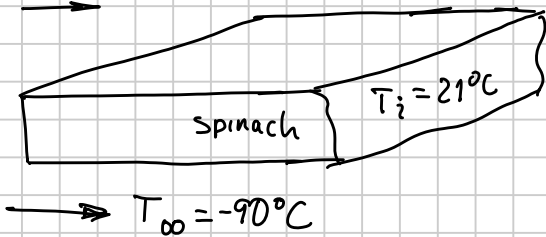
$$\dot{Q} = \dot{n} Q = (7.78 \text{ ball/s})(323.6) = 8989 \text{ kW}$$

Results and/or Analysis of Results

7.- A frozen-food company freezes its spinach by first compressing it into large slabs and then exposing the slab of spinach to a low-temperature cooling medium. The large slab of compressed spinach is initially at a uniform temperature of  $21^{\circ}\text{C}$ ; it must be reduced to an average temperature over the entire slab of  $-34^{\circ}\text{C}$ . The temperature at any part of the slab, however, must never drop below  $-51^{\circ}\text{C}$ . The cooling medium which passes across both sides of the slab is at a constant temperature of  $-90^{\circ}\text{C}$ . The following data may be used for the spinach: density =  $80 \text{ kg/m}^3$ ; thermal conductivity =  $0.87 \text{ W/(m K)}$ ; specific heat =  $2100 \text{ J/(kg K)}$ . Present a detailed analysis outlining a method estimate the maximum thickness of the slab of spinach that can be safely cooled in 60 min.

### Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)

$$T_{\infty} = -90^{\circ}\text{C}$$



$$\rho = 80 \text{ kg/m}^3$$

$$K = 0.8 \text{ W/mK}$$

$$C_p = 2100 \text{ J/kg K}$$

$$2L = ?$$

$$@ t = 60 \text{ min} = 3600 \text{ s}$$

$$T_{\text{avg}} = -34^{\circ}\text{C}$$

$$\alpha = \frac{K}{\rho C_p} = \frac{0.8}{80(2100)} = 4.762 \times 10^{-6} \text{ m}^2/\text{s}$$

Assuming spatial effect

$$T_{\text{avg}} = \frac{T_{\text{center}} + T_{\text{surface}}}{2}$$

assuming

$$T_{\text{surface}} \geq -51^{\circ}\text{C}$$

$$T_o = 2(-34) - (-51)$$

$$T_o = -17^{\circ}\text{C}$$

$$T_{\text{avg}} = -34^{\circ}\text{C}$$

### Knowledge (Equations that governing the phenomenon)

INFINITE WALL

$$\frac{T - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\frac{\lambda_1 x}{L}\right)$$

Using ONE TERM APPROX

### Application (Solution procedure, all computations)

for  $x = L$

$$\frac{T_L - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1)$$

$$\frac{-51 - (-90)}{21 - (-90)} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1)$$

$$\frac{39}{111} = A_1 e^{-\lambda_1^2 \tau} \cos \lambda_1 \quad \sim (2)$$

Sus. Eq (1) and Eq (2)

$$\frac{39}{111} = \frac{73}{111} \cos \lambda_1$$

$$\cos \lambda_1 = \frac{39}{73}$$

$$\lambda_1 = \cos^{-1}(0.5342)$$

$$\lambda_1 = 1.0072 \quad \rightarrow \quad A_1 = 1.1594$$

$$B_1 = 1.678$$

### Results and/or Analysis of Results

for Center of wall

$$\frac{T_o - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau}$$

$$\frac{-17 - (-90)}{21 - (-90)} = A_1 e^{-\lambda_1^2 \tau}$$

$$\frac{73}{111} = A_1 e^{-\lambda_1^2 \tau} \quad \sim (1)$$

Sus.  $A_1$  in Eq (1)

$$\frac{73}{111} = (1.1594) e^{-(1.0072)^2 \tau}$$

$$\tau = \frac{\ln(0.5672)}{-(1.0072)^2} = 0.5590$$

$$\tau = \frac{\alpha t}{L^2}$$

$$L = \sqrt{\frac{(4.762 \times 10^{-6})(3600)}{0.559}}$$

$$L = 0.1751 \text{ m}$$

$$\text{thickness} = 2L = 0.3502 \text{ m}$$



8.- An egg, can be assumed to be a 5-cm-diameter sphere having the thermal properties of water, is initially at a temperature of 4°C. It is immersed in boiling water at 100°C for 15 min. The heat transfer coefficient from the water to the egg may be assumed to be 1700 W/(m² K). What is the temperature of the egg center at the end of the cooking period?



Egg

Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)

$$r_o = 0.025 \text{ m}$$

$$h = 1700 \text{ W/m}^2 \text{ K}$$

$$T_\infty = 100^\circ \text{C}$$

$$T|_{r=0} = ? \text{ @ } t = 15 \text{ min} = 900 \text{ s}$$

$$T_i = 4^\circ \text{C}$$

Water @ 30°C

$$k = 0.618 \text{ W/mK}$$

$$\rho = 995 \text{ kg/m}^3$$

$$C_p = 4.178 \text{ kJ/kg K}$$

$$\alpha = 0.15 \times 10^{-6} \text{ m}^2/\text{s}$$

Knowledge (Equations that governing the phenomenon)

Lumped-capacity Criteria  $Bi = \frac{hL_c}{k}$ SPATIAL EFFECT  $\tau > 0.2$ 

ONE TERM APPROX

$$Bi = \frac{hL_c}{k} = \frac{1700(0.025/3)}{0.618}$$

$$Bi = 22.92 > 0.1 \therefore$$

HEAT TRANSFER WITH SPATIAL EFFECTS

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.15 \times 10^{-6})(900)}{(0.025)^2}$$

$$\tau = 0.216 > 0.2$$

SOLVING WITH ONE TERM APPROX.

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 \bar{e}^{-\lambda_1^2 \tau}$$

$$Bi = \frac{hr_o}{k} = \frac{(1700)(0.025)}{0.618} = 68.77$$

$$\hookrightarrow \begin{aligned} A_1 &= 1.9973 \\ \lambda_1 &= 3.0906 \end{aligned}$$

$$T_o = T_\infty + (T_i - T_\infty) A_1 \bar{e}^{-\lambda_1^2 \tau}$$

$$T_o = 100 + (4 - 100)(1.9973) e^{-(3.0906)^2 (0.216)}$$

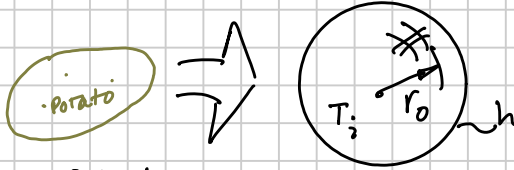
$$T_o = 75.64^\circ \text{C}$$

Application (Solution procedure, all computations)



9.- A potato weighing 0.25 kg is dropped into boiling water at 100°C. If the potato is considered spherical in shape, possesses the thermophysical properties of water, and is initially at 20°C, determine the time required for its center temperature reach 95°C. Assume heat transfer coefficient for boiling is very large. Also calculate the amount of heat loss.

### Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$m = 0.25 \text{ kg}$$

$$\text{Water @ } 20^\circ\text{C}$$

$$\rho = 998.0 \text{ kg/m}^3$$

$$c_p = 4182 \text{ J/kg K}$$

$$k = 0.598 \text{ W/m K}$$

$$T_\infty = 100^\circ\text{C}$$

$$T_i = 20^\circ\text{C}$$

$$h = 1000 \text{ W/m}^2\text{ }^\circ\text{C}$$

$$t = ? @ T|_{r=0} = 95^\circ\text{C}$$

$$Q = ?$$

$$\rho = \frac{m}{V} \quad V = \frac{m}{\rho}$$

$$r_0 = \sqrt[3]{\frac{3(0.25)}{4\pi(998.0)}} = 0.03911 \text{ m}$$

Assuming  $\tau > 0.2$   
In order to used One term Sol

$h \rightarrow \text{very large}$

$$\therefore Bi \rightarrow \infty$$

$$\lambda_1 = 3.1416$$

$$A_1 = 2.0$$

### Knowledge (Equations that governing the phenomenon)

SPATIAL EFFECT  
ONE TERM APPROX.

$$\frac{T - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

### Application (Solution procedure, all computations)

$$\alpha = \frac{k}{\rho c_p}$$

$$\alpha = \frac{0.598}{(998)(4182)} = 1.433 \times 10^{-7}$$

$$\frac{95 - 100}{20 - 100} = 3.1416 e^{-(3.1416)^2 \tau}$$

$$0.0625 = 2.000 e^{-3.1416^2 \tau}$$

$$0.03125 = e^{-3.1416^2 \tau}$$

$$\tau = 0.3512$$

$$\tau = \frac{\alpha t}{r_0^2}$$

$$t = \frac{0.3512(0.03911)^2}{(1.433 \times 10^{-7})}$$

$$t = 3749 \text{ s}$$

$$t = 62.5 \text{ min}$$

$$Q_{\max} = m c_p (T_\infty - T_i)$$

$$\frac{Q}{Q_{\max}} = 1 - 3 \theta_{0, \text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

$$\frac{Q}{Q_{\max}} = 1 - 3(0.0625) \frac{\sin \pi - \pi \cos \pi}{\pi^3}$$

$$\frac{Q}{Q_{\max}} = 1 - 0.1875 \left( \frac{1}{\pi^2} \right)$$

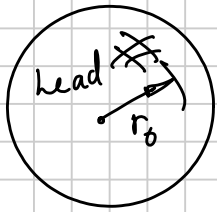
$$Q = (0.981)(0.25)(4182)(100 - 20)$$

$$Q = 82.05 \text{ kJ}$$

### Results and/or Analysis of Results

10.- Lead shot may be manufactured by dropping molten-lead droplets into water. Initially the droplets have the melting point temperature, calculate the time for the center temperature to reach  $120^\circ\text{C}$  when the water is at  $100^\circ\text{C}$  with  $h = 5000 \text{ W/m}^2\cdot^\circ\text{C}$ ,  $d = 15 \text{ mm}$ . Assuming that the droplets have the properties of solid lead at  $300 \text{ K}$ .

### Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$r_0 = 0.0075 \text{ m}$$

$$T_i = 601 \text{ K} = 328^\circ\text{C}$$

$$t = ? @ T_{r=0} = 120^\circ\text{C}$$

$$T_\infty = 100^\circ\text{C}$$

$$h = 5000 \text{ W/m}^2\cdot^\circ\text{C}$$

$$\alpha = 24.1 \times 10^{-6} \text{ m}^2/\text{s}$$

$$K = 35.3 \text{ W/m K}$$

$$C_p = 129 \text{ J/kg K}$$

$$\rho = 11340 \text{ kg/m}^3$$

$$h_c = \frac{V}{A} = \frac{r_0}{3} = \frac{0.0075}{3} = 2.5 \times 10^{-3}$$

$$Bi = \frac{h h_c}{K} = \frac{5000(2.5 \times 10^{-3})}{35.3} = 0.354$$

$Bi > 0.1 \therefore$  heat transfer with spatial effects

Assuming  $\tau > 0.2$  so

Using ONE TERM APPROX.

$$Bi = \frac{h r_0}{K} = \frac{5000(0.0075)}{35.3} = 1.062$$

$$\begin{aligned} A_1 &= 1.2828 \\ \lambda_1 &= 1.5992 \end{aligned}$$

### Knowledge (Equations that governing the phenomenon)

ONE TERM APPROX

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

### Application (Solution procedure, all computations)

$$\frac{120 - 100}{328 - 100} = 1.2828 e^{-1.5992^2 \tau}$$

$$\ln 0.06838 = -2.557 \tau$$

$$\tau = \frac{\ln 0.06838}{-2.5574} = 1.049$$

$$\tau = \frac{\alpha t}{r_0^2}$$

$$t = \frac{\tau r_0^2}{\alpha}$$

$$t = \frac{1.049(0.0075)^2}{24.1 \times 10^{-6}} = \underline{2.45 \text{ s}}$$

### Results and/or Analysis of Results

11.- The mechanical engineers at the Poly end winter term by roasting a pig and having a picnic. The pig is roughly cylindrical and about 26 cm in diameter. It is roasted over a propane flame, whose products have properties similar to those of air, at 280°C. The hot gas flows across the pig is at high velocity such as can assuming Biot number to be large. If the meat is cooked when it reaches 95°C, and if it is to be served at 2:00 pm, what time should cooking commence? The pig is initially at 25°C. The properties of lean pork are:  $k = 0.456 \text{ W/m}\cdot\text{K}$ ,  $C_p = 3490 \text{ J/kg}\cdot\text{K}$ , and  $\rho = 1030 \text{ kg/m}^3$

Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$r_0 = 0.13 \text{ m}$$

$$T_\infty = 280^\circ\text{C}$$

$$T_i = 25^\circ\text{C}$$

$$h \rightarrow \infty$$

what time start?

$$@ T/r_0 = 95^\circ\text{C}$$

$$k = 0.456 \text{ W/m}\cdot\text{K}$$

$$C_p = 3490 \text{ J/kg}\cdot\text{K}$$

$$\rho = 1030 \text{ kg/m}^3$$

$$\alpha = 0.13 \times 10^{-6} \text{ m}^2/\text{s}$$

$$h \rightarrow \infty$$

$$Bi \rightarrow \infty$$

assuming  $\tau > 0.2$

ONE TERM SOLUTION

for  $r=0$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$Bi \rightarrow \infty \rightarrow \begin{cases} \lambda_1 = 2.4048 \\ A_1 = 1.6021 \end{cases}$$

Knowledge (Equations that governing the phenomenon)

ONE TERM SOLUTION

for  $r=0$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$\tau = \frac{\alpha t}{r_0^2}$$

Application (Solution procedure, all computations)

$$\frac{95 - 280}{25 - 280} = 1.6021 e^{-2.4048^2 \tau}$$

$$0.45284 = e^{-5.7831 \tau}$$

$$\tau = \frac{\ln 0.45284}{-5.7831}$$

$$\tau = 0.1370$$

$$t = \frac{\tau r_0^2}{\alpha} = \frac{(0.1370)(0.13)^2}{0.13 \times 10^{-6}}$$

$$t = 17810 \text{ s}$$

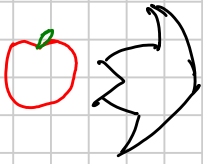
$$t = 4.947 \approx 5 \text{ h}$$

Start @ 9:00 AM

Results and/or Analysis of Results

12.- A dozen approximately spherical apples, 10 cm in diameter are taken from a 30°C environment and laid out on a rack in a refrigerator at 5°C,  $h$  is approximately 6 W/m<sup>2</sup>K as the result of natural convection. What will be the temperature of the centers of the apples after 70 mins? How much heat will the refrigerator have to carry away in this time?

### Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$r_o = 0.05 \text{ m}$$

$$T_i = 30^\circ\text{C}$$

$$T_\infty = 5^\circ\text{C}$$

$$h = 6 \text{ W/m}^2\text{K}$$

$$T|_{r=0} = ? \text{ @ } t = 4200 \text{ s}$$

$$Q = ?$$

Apple

$$\rho = 840 \text{ kg/m}^3$$

$$k = 0.418 \text{ W/mK}$$

$$\alpha = 0.13 \times 10^{-6} \text{ m}^2/\text{s}$$

$$C_p = 3.81 \text{ kJ/kg}\cdot\text{K}$$

for lumped Analysis

$$Bi = \frac{h L_c}{k} = \frac{6 (0.025)}{0.418} = 0.358$$

$$Bi > 0.1$$

∴ Heat transfer with spatial effects

$$Bi = \frac{h r_o}{k} = \frac{6 (0.05)}{0.418} = 0.7177$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{0.13 \times 10^{-6} (4200)}{0.05^2} = 0.2184$$

$$\tau > 0.2$$

ONE TERM APPROX SOL. CAN BE USED.

### Knowledge (Equations that governing the phenomenon)

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$Bi = \frac{h r_o}{k}$$

$$\tau = \frac{\alpha t}{r_o^2}$$

$$Bi = 0.7177$$

↳ from table 4-2

$$\lambda_1 = 1.3666$$

$$A_1 = 1.2024$$

### Application (Solution procedure, all computations)

$$T_o = T_\infty + (T_i - T_\infty) A_1 e^{-\lambda_1^2 \tau}$$

$$T_o = 5 + (30 - 5) (1.2024) e^{-(1.3666)^2 (0.2184)}$$

$$T_o = 5 + (25) (0.7997)$$

$$T_o = 25^\circ\text{C}$$

$$\frac{Q}{Q_{\max}} = 1 - 3 \theta_{0, \text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

$$Q_{\max} = m C_p (T_i - T_\infty)$$

$$Q_{\max} = 12 \left[ (840) \frac{4}{3} \pi (0.05)^3 (3.81) (30 - 5) \right]$$

$$Q_{\max} = 502.7 \text{ kJ}$$

$$\frac{Q}{Q_{\max}} = 1 - 3 \left( \frac{25 - 5}{30 - 5} \right) \frac{\sin 1.3666 - 1.3666 \cos 1.3666}{1.3666^3}$$

$$\frac{Q}{Q_{\max}} = 0.3398$$

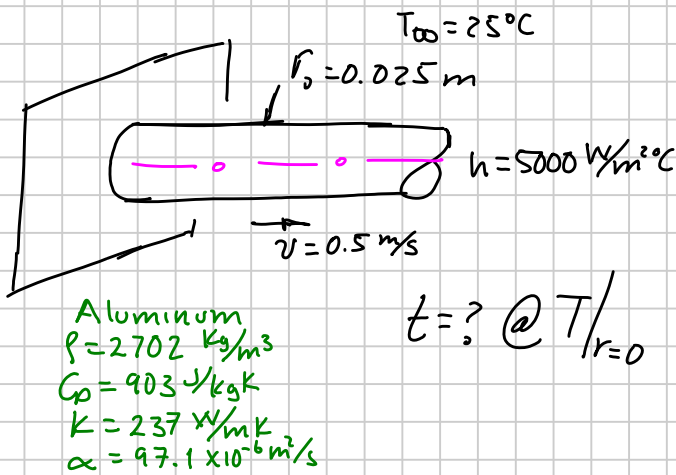
$$Q = 0.3398 (502.7)$$

$$Q = 170.8 \text{ kJ}$$

### Results and/or Analysis of Results

13.- In an aluminum factory, aluminum cylinder bars of 5 cm diameter are extruded at 500 °C. An “infinite” extruded bar passes through a water spray bath after it leaves the extrusion die. The water spray is available at a temperature of 25 °C. The heat transfer coefficient between the cooling water and the surface of the bars is 5000 W/m<sup>2</sup> °C. If the bars are extruded at a velocity of 0.5 m/s, determine the length  $L$  of the cooling tank required to reduce the centerline temperature of the bars to 150°C. (hint:  $L = \text{velocity} \times \text{time}$ )

### Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



Lumped Verification

$$L_c = \frac{r_o}{2} = 0.0125 \text{ m}$$

$$Bi = \frac{h L_c}{k} = \frac{5000 (0.0125)}{237} = 0.2637 > 0.1$$

∴ spatial effects  
 $T(r, t)$

Assuming  $\gamma > 0.2$

to Apply ONE TERM SOLUTION

### Knowledge (Equations that governing the phenomenon)

$$\frac{T - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \gamma}$$



$$Bi = \frac{h r_o}{k} = \frac{5000 (0.025)}{237}$$

$$\gamma = \frac{\alpha t}{r_o^2}$$

$$\gamma = \frac{97.1 \times 10^{-6} t}{0.025^2}$$

$$Bi = 0.5274$$

$$A_1 = 1.1144$$

$$\lambda_1 = 0.9621$$

$$\gamma = 0.1554 t$$

### Application (Solution procedure, all computations)

$$\frac{150 - 25}{500 - 25} = 1.1144 e^{-0.9621^2 \gamma}$$

$$\gamma = -\frac{1}{0.9621^2} \ln 0.2361$$

$$\gamma = 1.559$$

$$t = \frac{1.559}{0.1554}$$

$$t = 10.03 \text{ s}$$

$$L = v t$$

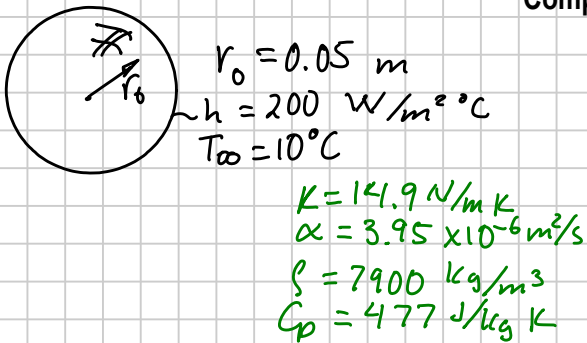
$$L = (0.5)(10.03)$$

$$L = 5.015 \text{ m}$$

### Results and/or Analysis of Results

14.- A stainless steel (AISI 304) sphere 10 cm OD is originally at a temperature of 500°C it suddenly immersed in a liquid at 10 °C for which the convective heat transfer coefficient is 200 W/m<sup>2</sup>°C. Determine the time required for the center of sphere to reach a temperature of 200°C.

Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$L_c = \frac{V}{A_s} = \frac{r_o}{3} = \frac{0.05}{3} = \frac{1}{60} = 0.01667 \text{ m}$$

$$Bi = \frac{h L_c}{k} = \frac{200(0.01667)}{14.9} = 0.2236 > 0.1$$

Spatial Effect

$$Bi = \frac{h r_o}{k} = \frac{200(0.05)}{14.9}$$

$$Bi = 0.6711$$

Assuming  $\tau > 0.2$  USING ONE TERM SOLUTION

from table 4-2

$$\lambda_1 = 1.3270$$

$$A_1 = 1.1901$$

Knowledge (Equations that governing the phenomenon)

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$Bi = \frac{h r_o}{k}$$

$$\tau = \frac{\alpha t}{r_o^2}$$

Application (Solution procedure, all computations)

$$\frac{Q}{Q_{\max}} = 1 - 3 \theta_{0, \text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

$$Q_{\max} = m C_p (T_i - T_\infty)$$

$$Q_{\max} = (7900) \frac{4}{3} \pi (0.05)^3 (477) (500 - 10)$$

$$Q_{\max} = 966.8 \text{ kJ}$$

$$\frac{Q}{Q_{\max}} = 1 - 3 \left( \frac{200 - 10}{500 - 10} \right) \frac{\sin 1.327 - 1.327 \cos 1.327}{1.327^3}$$

$$= 1 - 3(0.3878)(0.2782)$$

$$\frac{Q}{Q_{\max}} = 0.6763$$

$$Q = 0.6763 (966.8)$$

$$Q = 653.9 \text{ kJ}$$

$$\frac{200 - 10}{500 - 10} = 1.1901 e^{-1.327^2 \tau}$$

$$\ln \left( \frac{0.3878}{1.1901} \right) = -1.327^2 \tau$$

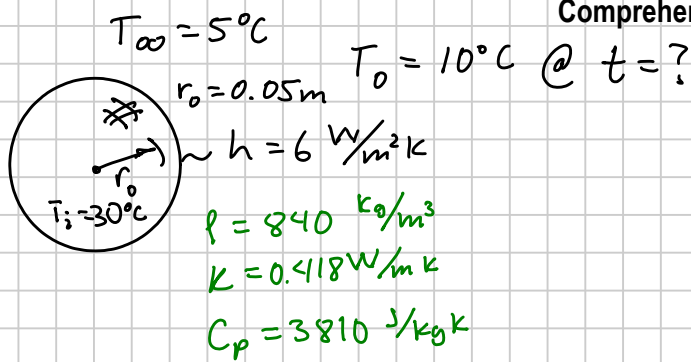
$$\tau = 0.6368$$

$$t = \frac{0.6368 (0.05)^2}{3.95 \times 10^{-6}}$$

$$t = 403.0 \text{ s} \quad / \quad 7.2 \text{ min}$$

Results and/or Analysis of Results

15.- A dozen approximately spherical apples, 10 cm in diameter are taken from a 30°C environment and laid out on a rack in a refrigerator at 5°C,  $h$  is approximately 6 W/m<sup>2</sup>K as the result of natural convection. How long will it take to bring the centers to 10°C? How much heat will the refrigerator have to carry away to get the centers to 10°C?



Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)

Assuming  $\tau > 0.2$  (check later)

One term Approximation

$$Bi = \frac{h r_o}{k} = \frac{6(0.05)}{0.418} = 0.7177$$

$$Bi = 0.7177$$

$$\lambda_1 = 1.3666$$

$$A_1 = 1.2024$$

Knowledge (Equations that governing the phenomenon)

$$\frac{T_o - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \quad Bi = \frac{h r_o}{k}$$

$$\alpha = \frac{k}{\rho C_p} \quad \tau = \frac{\alpha t}{r_o^2}$$

Application (Solution procedure, all computations)

Heat loss

$$Q_{\max} = m C_p (T_i - T_{\infty})$$

$$Q_{\max} = (840) \left( \frac{4}{3} \pi \right) (0.05)^3 (3810) (30 - 5)$$

$$Q_{\max} = 41.89 \times 10^3 \text{ J}$$

$$Q = Q_{\max} \left( 1 - 3 \theta_0 \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3} \right)$$

$$Q = 12 (41.89 \times 10^3) \left[ 1 - 3(0.2) \frac{\sin(1.3666) - 1.3666 \cos(1.3666)}{1.3666^3} \right]$$

$$Q = 502.7 \times 10^3 (0.8350)$$

$$Q = 419.8 \text{ kJ}$$

$$\frac{10 - 5}{30 - 5} = 1.2024 e^{-(1.3666)^2 \tau}$$

$$0.2 = 1.2024 e^{-1.8676 \tau}$$

$$\tau = \frac{\ln 0.16633}{-1.8676}$$

$$\tau = 0.9605 > 0.2 \quad \checkmark$$

$$t = \frac{\tau r_o^2}{\alpha}$$

$$t = \frac{0.9605 (0.05)^2}{1.306 \times 10^{-7}}$$

$$t = 18386 \text{ s}$$

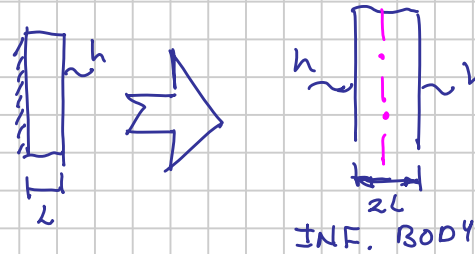
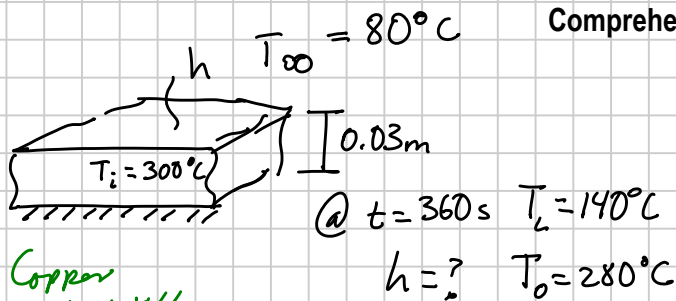
$$t = 5.1 \text{ hrs.}$$

Results and/or Analysis of Results



16.- A large slab of copper having a thickness of 3.0 cm is initially at 300 °C. It is suddenly exposed to a convection environment on the top surface at 80 °C while the bottom surface is insulated. In 6 min the surface temperature drops to 140 °C and the insulated surface drops to 280 °C. Calculate the value of the convection heat-transfer coefficient.

Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



Copper  
 $k = 401 \text{ W/mK}$   
 $\rho = 8933 \text{ kg/m}^3$   
 $c_p = 385 \text{ J/kgK}$   
 $\alpha = 117 \times 10^{-6} \text{ m}^2/\text{s}$

$$\tau = \frac{\alpha t}{L} = \frac{(117 \times 10^{-6})(360)}{0.03}$$

$$\tau = 1.404$$

$$@ t = 360 \text{ s}$$

$$\frac{T_o - T_{\infty}}{T_i - T_{\infty}} = \frac{280 - 80}{300 - 80} = A_1 e^{-\lambda_1^2 \tau}$$

$$\frac{T_L - T_{\infty}}{T_i - T_{\infty}} = \frac{140 - 80}{300 - 80} = A_1 e^{-\lambda_1^2 \tau} \cos \lambda_1$$

Knowledge (Equations that governing the phenomenon)

$$Bi = \frac{hL}{k} \quad \text{One-term}$$

$$\text{for center } \theta_{o, \text{wall}} = \frac{T_o - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau}$$

$$\text{for surface } \theta_{L, \text{wall}} = \frac{T_L - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \cos \lambda_1$$

Application (Solution procedure, all computations)

$$\frac{10}{11} = A_1 e^{-\lambda_1^2 \tau} \quad (1)$$

$$\frac{3}{11} = A_1 e^{-\lambda_1^2 \tau} \cos \lambda_1 \quad (2)$$

$$\frac{3}{10} = \cos \lambda_1$$

$$\cos \lambda_1 = 0.3$$

$$\lambda_1 = 1.2661$$

from table 4-2

$$Bi = 4.031$$

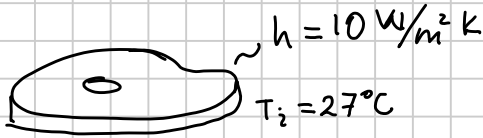
$$h = \frac{Bi k}{L} = \frac{(4.031)(401)}{0.03}$$

$$h = 53,881 \text{ W/m}^2 \cdot ^{\circ}\text{C}$$

Results and/or Analysis of Results

17.- In the inspection of a sample of meat intended for human consumption, it was found that certain undesirable organisms were present. In order to make the meat safe for consumption, it is ordered that the meat be kept at a temperature of at least  $121^\circ\text{C}$  for a period of at least 20 min during the preparation. Assume that a 2.5-cm-thick slab of this meat is originally at a uniform temperature of  $27^\circ\text{C}$ ; that it is to be heated from both sides in a constant temperature oven; and that the maximum temperature meat can withstand is  $154^\circ\text{C}$ . Assume furthermore that the surface coefficient of heat transfer remains constant and is  $10\text{ W}/(\text{m}^2\text{ K})$ . The following data may be taken for the sample of meat: specific heat =  $4184\text{ J}/(\text{kg K})$ ; density =  $1280\text{ kg}/\text{m}^3$ ; thermal conductivity =  $0.45\text{ W}/(\text{m K})$ . Calculate the oven temperature and the minimum total time of heating required to fulfill the safety

### Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$2L = 0.025\text{ m}$$

$$C_p = 4184\text{ J}/(\text{kg K})$$

$$\rho = 1280\text{ kg}/\text{m}^3$$

$$k = 0.48\text{ W}/(\text{m K})$$

$$T_{\max} = 154^\circ\text{C}$$

$$T_{\min} = 121^\circ\text{C} \quad t_{\min} = 20\text{ min}$$

$$T_\infty = ? \quad \& \quad t = ?$$

$$\alpha = \frac{k}{\rho C_p} = \frac{0.48}{1280(4184)} = 8.963 \times 10^{-8}\text{ m}^2/\text{s}$$

$$B_i = \frac{hL}{k}$$

$$B_i = \frac{(10)(0.0125)}{(0.48)} = 0.2604$$

$$\lambda_1 = 0.4866$$

$$A_1 = 1.0395$$

| $B_i$ | $\lambda_1$ | $A_1$  |
|-------|-------------|--------|
| 0.2   | 0.4328      | 1.0311 |
| 0.3   | 0.5218      | 1.0450 |

### Knowledge (Equations that governing the phenomenon)

INFINITE WALL

$$\text{@ } x=0 \quad \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \quad \tau = \frac{\alpha t}{L^2}$$

$$\text{@ } x=L$$

$$\frac{T_L - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos \lambda_1$$

Requirement

@ center in  $t_c = t - t_{\min}$

$$\tau_c = \frac{8.963 \times 10^{-8} (t - 1200)}{0.0125^2} = 5.736 \times 10^{-4} (t - 1200)$$

$$\tau_c = 5.736 \times 10^{-4} (t - 1200)$$

$$\frac{121 - T_\infty}{27 - T_\infty} = 1.0395 e^{-0.4866^2 \tau_c}$$

$$(121 - T_\infty) = (27 - T_\infty) (1.0395) e^{-1.358 \times 10^{-4} (t - 1200)}$$

$$(121 - T_\infty) = (27 - T_\infty) (1.0395) e^{-1.358 \times 10^{-4} t} e^{0.1630} \quad e^{0.1630} \sim (1)$$

### Application (Solution procedure, all computations)

@ surface  $T_{\max}$

$$\tau = \frac{8.963 \times 10^{-8} t}{0.0125^2} = 5.736 \times 10^{-4} t$$

$$\frac{154 - T_\infty}{27 - T_\infty} = 1.0395 e^{-(0.4866)^2 \tau} \cos 0.4866$$

$$(154 - T_\infty) = (0.8839) (27 - T_\infty) (1.0395) e^{-1.358 \times 10^{-4} t} \quad \sim (2)$$

Subst  $E_q(1)$  in  $E_q(2)$

$$(154 - T_\infty) = (0.8839) \frac{(121 - T_\infty)}{e^{0.1630}}$$

$$154 - T_\infty = 0.7510 (121 - T_\infty)$$

$$T_\infty = \frac{154 - (0.7510)(121)}{(1 - 0.7510)}$$

$$T_\infty = 253.5^\circ\text{C}$$

$$T_\infty = 253.5^\circ\text{C}$$

Subst in  $E_q(2)$

$$-99.5 = (0.8839)(-226.5)(1.0395) e^{-1.358 \times 10^{-4} t}$$

$$0.4781 = e^{-1.358 \times 10^{-4} t}$$

$$t = \frac{\ln 0.4781}{-1.358 \times 10^{-4}}$$

$$t = 5434\text{ s}$$

$$t = 90.6\text{ min}$$

$$\tau = 3.117 > 0.2$$

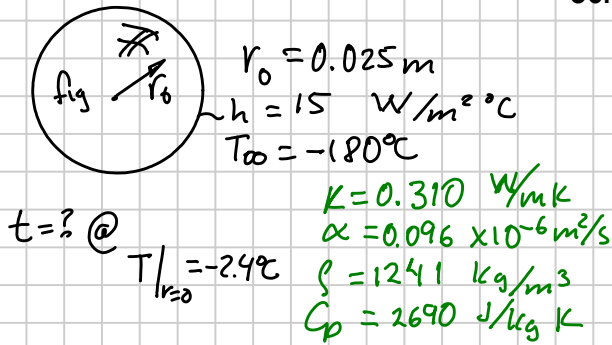
$$T_0 = 253.5 + (27 - 253.5) 1.0395 e^{-(0.4866)^2 (3.117)}$$

$$T_0 = 141^\circ\text{C} \quad \text{@}$$

### Results and/or Analysis of Results

18.- Fresh figs fruit (with diameter,  $D = 0.05$  m) are initially at  $20^\circ\text{C}$  and must be individually frozen by suddenly placing them in cold nitrogen gas at  $-180^\circ\text{C}$ . The freezing temperature for the figs is  $-2.4^\circ\text{C}$ . The convective heat transfer coefficient between the figs and the gaseous nitrogen is  $15$   $\text{W/m}^2\text{-K}$  for natural convection. Determine the time required to cool the figs center to its freezing temperature.

## Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$L_c = \frac{V}{A_s} = \frac{r_o}{3} = \frac{0.025}{3} = \frac{1}{120} = 0.008333 \text{ m}$$

$$Bi = \frac{h L_c}{k} = \frac{15 (0.008333)}{0.310} = 0.4032 > 0.1$$

Spatial Effect

$$Bi = \frac{h r_o}{k} = \frac{15 (0.025)}{0.310}$$

$$Bi = 1.210$$

## Knowledge (Equations that governing the phenomenon)

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \quad Bi = \frac{h r_o}{k}$$

$$\tau = \frac{\alpha t}{r_o^2}$$

Assuming  $\tau > 0.2$   $\therefore$  USING ONE TERM SOLUTION

$Bi$  from table 4-2

$$\lambda_1 = 1.6670$$

$$A_1 = 1.3165$$

## Application (Solution procedure, all computations)

$$\frac{-2.4 - (-180)}{20 - (-180)} = 1.3165 e^{-1.667^2 \tau}$$

$$\ln\left(\frac{0.888}{1.3165}\right) = -1.667^2 \tau$$

$$\tau = 0.1417 < 0.2 \quad \text{NOTE}$$

$$t = \frac{0.1417 (0.025)^2}{0.096 \times 10^{-6}}$$

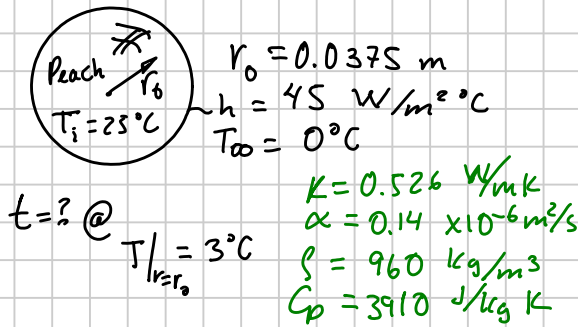
$$t = 922.5 \text{ s} \quad 15.4 \text{ min}$$

NOTE: the solution may have an error greater than 2%

## Results and/or Analysis of Results

19.- Peaches with a diameter of about 75 mm are to be cooled from room temperature of 25°C to 3°C using an air-convection environment with  $h = 45 \text{ W/m}^2\cdot^\circ\text{C}$  and  $T_\infty = 0^\circ\text{C}$ . Calculate the time required for the cooling the peaches surface to 3°C and the total cooling required for 100 peaches.

Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$L_c = \frac{V}{A_s} = \frac{r_o}{3} = \frac{0.0375}{3} = \frac{1}{80} = 0.0125$$

$$Bi = \frac{h L_c}{k} = \frac{45 (0.0125)}{0.526} = 1.069 > 0.1$$

Spatial Effect

$$Bi = \frac{h r_o}{k} = \frac{45 (0.0375)}{0.526}$$

$$Bi = 3.208$$

Assuming  $\tau > 0.2$   $\therefore$  USING ONE TERM SOLUTION

from table 4-2

$$\lambda_1 = 2.3236$$

$$A_1 = 1.6430$$

$$\frac{3 - 0}{25 - 0} = 1.6430 e^{-2.3236^2 \tau} \frac{\sin 2.3236}{2.3236}$$

$$\ln \left( \frac{0.12}{0.5160} \right) = -5.399 \tau$$

$$\tau = 0.2702 < 0.2$$

$$t = \frac{0.2702 (0.0375)^2}{0.14 \times 10^{-6}}$$

$$t = 2714 \text{ s} \quad 45.3 \text{ min}$$

Knowledge (Equations that governing the phenomenon)

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin \lambda_1}{\lambda_1}$$

$$Bi = \frac{h r_o}{k}$$

$$\tau = \frac{\alpha t}{r_o^2}$$

Application (Solution procedure, all computations)

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \theta_o = A_1 e^{-\lambda_1^2 \tau}$$

$$\theta_o = 1.6430 e^{-2.3236^2 (0.2702)}$$

$$\theta_o = 0.3820$$

$$Q_{\max} = m C_p (T_i - T_\infty)$$

$$Q_{\max} = (960) \left( \frac{4}{3} \pi \right) (0.0375)^3 (3910) (25 - 0)$$

$$Q_{\max} = 20.73 \text{ kJ}$$

$$Q = Q_{\max} \left( 1 - 3 \theta_o \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3} \right)$$

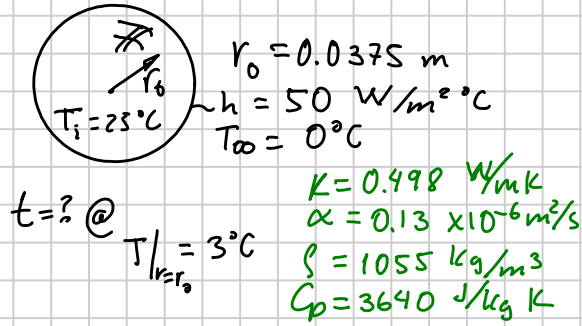
$$Q = 100 \left\{ (20.73) \left[ 1 - 3 (0.382) \frac{\sin(2.3236) - 2.3236 \cos(2.3236)}{2.3236^3} \right] \right\}$$

$$Q = 1.634 \text{ MJ}$$

Results and/or Analysis of Results

20.- A potato with a diameter of about 75 mm are to be cooled from room temperature of 25°C to 3°C using an air-convection environment with  $h = 50 \text{ W/m}^2\cdot^\circ\text{C}$  and  $T_\infty = 0^\circ\text{C}$ . Calculate the time required for the cooling the potato surface to 3°C and the total cooling required for 100 potatoes.

Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$L_c = \frac{V}{A_s} = \frac{r_o}{3} = \frac{0.0375}{3} = \frac{1}{80} = 0.0125$$

$$Bi = \frac{h L_c}{k}$$

$$Bi = \frac{50 (0.0125)}{0.498}$$

$$Bi = 1.255 > 0.1$$

Spatial Effect

$$Bi = \frac{h r_o}{k} = \frac{50 (0.0375)}{0.498}$$

$$Bi = 3.765$$

Assuming  $\tau > 0.2$  USING ONE TERM SOLUTION

from table 4-2

$$\lambda_1 = 2.4164$$

$$A_1 = 1.6973$$

$$\frac{3 - 0}{25 - 0} = 1.6973 \mathcal{C}^{-2.4164^2 \tau} \frac{\sin(2.4164)}{2.4164}$$

$$\ln\left(\frac{0.12}{0.4662}\right) = -5.839 \tau$$

$$\tau = 0.2324 < 0.2$$

$$t = \frac{0.2324 (0.0375)^2}{0.13 \times 10^{-6}}$$

$$t = 2514 \text{ s} \quad 41.90 \text{ min}$$

Knowledge (Equations that governing the phenomenon)

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 \mathcal{C}^{-\lambda_1^2 \tau} \frac{\sin \lambda_1}{\lambda_1} \quad Bi = \frac{h r_o}{k} \quad \tau = \frac{\alpha t}{r_o^2}$$

Application (Solution procedure, all computations)

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \theta_o = A_1 \mathcal{C}^{-\lambda_1^2 \tau}$$

$$\theta_o = 1.6973 \mathcal{C}^{-2.4164^2 (0.2324)}$$

$$\theta_o = 0.4370$$

$$Q_{\max} = m C_p (T_i - T_\infty)$$

$$Q_{\max} = (1055) \left(\frac{4}{3} \pi\right) (0.0375)^3 (3640) (25 - 0)$$

$$Q_{\max} = 21.21 \text{ kJ}$$

$$Q = Q_{\max} \left(1 - 3 \theta_o \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}\right)$$

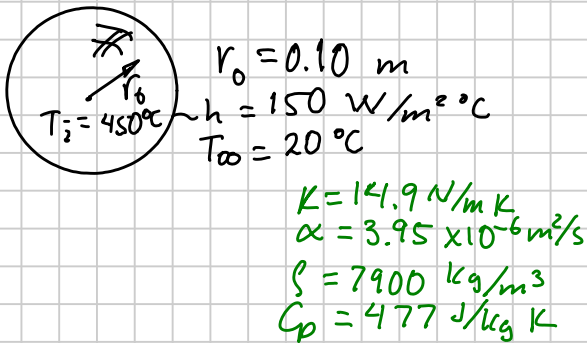
$$Q = 100 \left\{ (21.21) \left[ 1 - 3 (0.4370) \frac{\sin(2.4164) - 2.4164 \cos(2.4164)}{2.4164^3} \right] \right\}$$

$$Q = 1.634 \text{ MJ}$$

Results and/or Analysis of Results

21.- A stainless steel (AISI 304) sphere 20 cm OD is originally at a temperature of 450 °C it suddenly immersed in a liquid at 20 °C for which the convective heat transfer coefficient is 150 W/m<sup>2</sup> °C. Determine the time required for the center of sphere to reach a temperature of 200 °C. Also estimate the amount of heat transfer.

## Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$L_c = \frac{V}{A_s} = \frac{r_o}{3} = \frac{0.10}{3} = \frac{1}{30} = 0.0333 \text{ m}$$

$$Bi = \frac{h L_c}{k}$$

$$Bi = \frac{150(0.0333)}{14.9}$$

$$Bi = 0.3355 > 0.1$$

Spatial Effect

$$Bi = \frac{h r_o}{k} = \frac{150(0.10)}{14.9}$$

|     | $\lambda$ | $A_1$  |
|-----|-----------|--------|
| 1 - | 1.5708    | 1.2732 |
| 2 - | 2.0288    | 1.4793 |

Assuming  $\tau > 0.2$  USING ONE TERM SOLUTION

$$Bi = 1.007$$

from table 4-2

$$\lambda_1 = 1.5740$$

$$A_1 = 1.2746$$

## Knowledge (Equations that governing the phenomenon)

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \quad Bi = \frac{h r_o}{k}$$

$$\tau = \frac{\alpha t}{r_o^2}$$

## Application (Solution procedure, all computations)

$$\frac{Q}{Q_{\max}} = 1 - 3 \theta_{0, \text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

$$Q_{\max} = m c_p (T_i - T_\infty)$$

$$Q_{\max} = (7900) \frac{4}{3} \pi (0.10)^3 (477) (450 - 20)$$

$$Q_{\max} = 6.787 \text{ MJ}$$

$$\frac{Q}{Q_{\max}} = 1 - 3 \left( \frac{200 - 20}{450 - 20} \right) \frac{\sin 1.574 - 1.574 \cos 1.574}{1.574^3}$$

$$= 1 - 3(0.4186)(0.2577)$$

$$\frac{Q}{Q_{\max}} = 0.6764$$

$$\frac{200 - 20}{450 - 20} = 1.275 e^{-1.574^2 \tau}$$

$$\ln \left( \frac{0.4186}{1.2746} \right) = -1.574^2 \tau$$

$$\tau = 0.4494$$

$$t = \frac{0.4494(0.10)^2}{3.95 \times 10^{-6}}$$

$$t = 1138 \text{ s} \quad \underline{\underline{\approx 19 \text{ min}}}$$

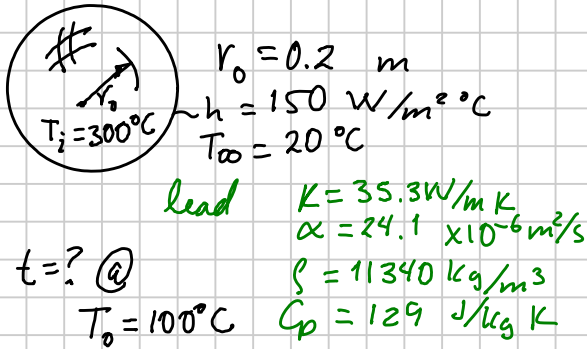
$$Q = 0.6764 (6.787 \times 10^6)$$

$$\underline{\underline{Q = 4.591 \text{ MJ}}}$$

## Results and/or Analysis of Results

22.- A lead sphere with diameter of 40 cm is originally at a temperature of 300 °C it suddenly immersed in a liquid at 20 °C for which the convective heat transfer coefficient is 150 W/m<sup>2</sup> °C. Determine the time required for the center of sphere to reach a temperature of 100 °C. Also estimate the amount of heat transfer.

### Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$L_c = \frac{V}{A_s} = \frac{r_o}{3} = \frac{0.20}{3} = \frac{1}{15} = 0.06667 \text{ m}$$

$$Bi = \frac{h L_c}{k}$$

$$Bi = \frac{150 (0.06667)}{35.3}$$

$$Bi = 0.2833 > 0.1$$

Spatial Effect

$$Bi = \frac{h r_o}{k} = \frac{150 (0.20)}{35.3}$$

|               | $\lambda$    | $A_1$  |
|---------------|--------------|--------|
| $Bi = 0.8499$ | 0.8 - 1.4320 | 1.2236 |
|               | 0.9 - 1.5044 | 1.2488 |

Assuming  $\tau > 0.2$

from table 4-2

$$\lambda_1 = 1.4681$$

$$A_1 = 1.2362$$

USING  
ONE TERM  
SOLUTION

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$Bi = \frac{h r_o}{k}$$

$$\tau = \frac{\alpha t}{r_o^2}$$

$$\frac{Q}{Q_{\max}} = 1 - 3 \theta_{0, \text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

$$Q_{\max} = m C_p (T_i - T_\infty)$$

### Application (Solution procedure, all computations)

$$\frac{100 - 20}{300 - 20} = 1.2362 e^{-1.4681^2 \tau}$$

$$\ln \left( \frac{0.2857}{1.2362} \right) = -1.4681^2 \tau$$

$$\tau = \frac{-1.465}{-2.155}$$

$$\tau = 0.6797 > 0.2$$

$$\tau = \frac{\alpha t}{r_o^2}$$

$$t = \frac{0.6797 (0.20)^2}{24.1 \times 10^{-6}}$$

$$t = 1128 \text{ s} \quad \underline{\underline{\approx 18.8 \text{ min}}}$$

$$Q_{\max} = (11340) \frac{4\pi (0.20)^3}{3} (129) (300 - 20)$$

$$Q_{\max} = 13.73 \text{ MJ}$$

$$\frac{Q}{Q_{\max}} = 1 - 3 \left( \frac{100 - 20}{300 - 20} \right) \frac{\sin 1.4681 - 1.4681 \cos 1.4681}{1.4681^3}$$

$$= 1 - 3(0.2857)(0.2664)$$

$$\frac{Q}{Q_{\max}} = 0.7717$$

$$Q = 0.7717 (13.73 \times 10^6)$$

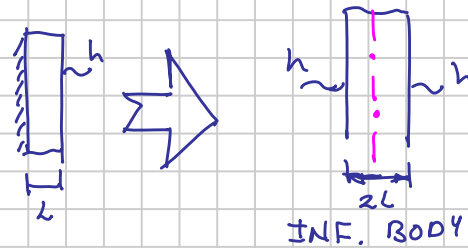
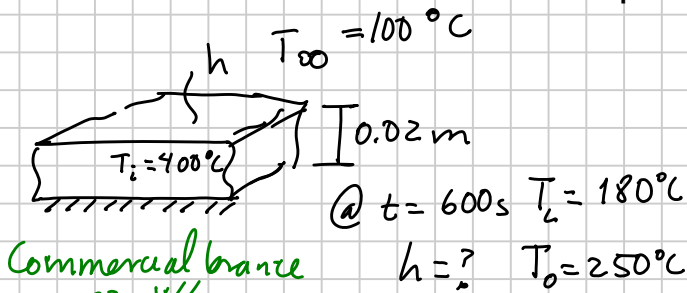
$$Q = 10.60 \text{ MJ}$$

### Results and/or Analysis of Results



23.- A large plate of commercial bronze having a thickness of 20 cm is initially at 400 °C. It is suddenly exposed to a convection environment on the top surface at 100 °C while the bottom surface is insulated. After 10 min the surface temperature drops to 180 °C and the insulated surface drops to 250°C. Calculate the value of the convection heat-transfer coefficient.

Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$\tau = \frac{\alpha t}{L} = \frac{(14 \times 10^{-6})(600)}{0.02}$$

$$\tau = 0.42$$

$$@ t = 600 \text{ s}$$

$$\frac{T_o - T_{\infty}}{T_i - T_{\infty}} = \frac{250 - 100}{400 - 100} = A_1 e^{-\lambda_1^2 \tau}$$

$$\frac{T_L - T_{\infty}}{T_i - T_{\infty}} = \frac{180 - 100}{400 - 100} = A_1 e^{-\lambda_1^2 \tau} \cos \lambda_1$$

$$\frac{1}{2} = A_1 e^{-\lambda_1^2 \tau} \quad (1)$$

$$\frac{4}{15} = A_1 e^{-\lambda_1^2 \tau} \cos \lambda_1 \quad (2)$$

$$\frac{4}{15} = \frac{1}{2} \cos \lambda_1$$

$$\cos \lambda_1 = \frac{8}{15}$$

$$\lambda_1 = 1.008$$

from table 4-2

$$B_i = 1.682$$

$$h = \frac{B_i K}{L} = \frac{(1.682)(52)}{0.02}$$

$$h = 4373 \text{ W/m}^2 \text{ } ^\circ\text{C}$$

Knowledge (Equations that governing the phenomenon)

$$Bi = \frac{hL}{K} \quad \text{One-Term}$$

$$\text{for center } \theta_{o, \text{wall}} = \frac{T_o - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau}$$

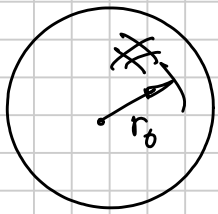
$$\text{for surface } \theta_{L, \text{wall}} = \frac{T_L - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1)$$

Application (Solution procedure, all computations)

Results and/or Analysis of Results

24.- How long does it take for a 5 cm diameter sphere of ice to start melting in whisky? Consider the temperature of the ice and whisky to be  $-8^{\circ}\text{C}$  and  $15^{\circ}\text{C}$ , respectively, when the convection coefficient is  $45 \text{ W/m}^2 \text{ }^{\circ}\text{C}$ .

Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



$$r_o = 0.025 \text{ m}$$

$$T_i = -8^{\circ}\text{C}$$

$$t = ? @ T|_{r=r_o} = 0^{\circ}\text{C}$$

$$T_{\infty} = 15^{\circ}\text{C}$$

$$h = 45 \text{ W/m}^2 \text{ }^{\circ}\text{C}$$

$$\text{ICE @ } 273 \text{ K}$$

$$k = 1.88 \text{ W/m K}$$

$$C_p = 2040 \text{ J/kg K}$$

$$\rho = 920 \text{ kg/m}^3$$

$$h_c = \frac{V}{A} = \frac{r_o}{3} = \frac{0.025}{3} = \frac{1}{120} \text{ m}$$

$$Bi = \frac{h_c}{k} = \frac{45 \left( \frac{1}{120} \right)}{1.88} = 0.1995$$

$Bi > 0.1 \therefore$  heat transfer with spatial effects

Assuming  $\tau > 0.2$  so

Using ONE TERM APPROX.

$$Bi = \frac{hr_o}{k} = \frac{45(0.025)}{1.88} = 0.5984$$

$$\begin{aligned} A_1 &= 1.1708 \\ \lambda_1 &= 1.2628 \end{aligned}$$

Knowledge (Equations that governing the phenomenon)

ONE TERM APPROX

$$\frac{T_r - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin \lambda_1}{\lambda_1}$$

Application (Solution procedure, all computations)

$$\frac{0 - 15}{-8 - 15} = 1.1708 e^{-1.2628^2 \tau} \frac{\sin(1.2628)}{1.2628}$$

$$\alpha = \frac{k}{\rho C_p}$$

$$\alpha = \frac{1.88}{(920)(2040)} = 1.002 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\ln(0.7382) = -3.399 \tau$$

$$\tau = \frac{\ln(0.7382)}{-1.5947} = 0.1903$$

$$\tau = \frac{\alpha t}{r_o^2}$$

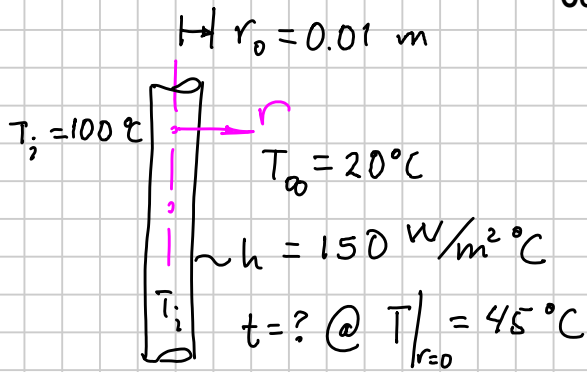
$$t = \frac{\tau r_o^2}{\alpha}$$

$$t = \frac{0.1903(0.025)^2}{1.002 \times 10^{-6}} = \underline{118.7 \text{ s}}$$

Results and/or Analysis of Results

25.- A long pyroceram rod with a diameter of 2 cm is initially at a uniform temperature of 100 °C. It is now exposed to an air stream at 20°C with a heat transfer coefficient of 150 W/m<sup>2</sup>·K. How long would the center rod be cooled to

Comprehension (State Definition, Free Body Diagram and/or Diagram Sketch)



Pyroceram

$$\rho = 2600 \text{ kg/m}^3$$

$$C_p = 808 \text{ J/kg K}$$

$$k = 3.98 \text{ W/m K}$$

$$\alpha = 1.89 \times 10^{-6} \text{ m}^2/\text{s}$$

Knowledge (Equations that governing the phenomenon)

ONE TERM APPROX

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$Bi = \frac{h r_o}{k}$$

$$h_c = \frac{r_o}{2} = \frac{0.01}{2} = 0.005 \text{ m}$$

$$Bi = \frac{150 (0.005)}{3.98} = 0.1884 > 0.1$$

$Bi > 0.1$   $\therefore$  heat transfer with spatial effects

Assuming  $\tau > 0.2$  so

Using ONE TERM APPROX.

$$Bi = \frac{h r_o}{k} = \frac{150 (0.01)}{3.98} = 0.3769$$

$$\begin{aligned} A_1 &= 1.088 \\ \lambda_1 &= 0.8273 \end{aligned}$$

Application (Solution procedure, all computations)

$$\frac{45 - 20}{100 - 20} = 1.088 e^{-(0.8273)^2 \tau}$$

$$\ln 0.2872 = -(0.8273)^2 \tau$$

$$\tau = \frac{\ln(0.2872)}{(0.8273)^2}$$

$$\tau = 1.823$$

$$\tau = \frac{\alpha t}{r_o^2}$$

$$t = \frac{\tau r_o^2}{\alpha}$$

$$t = \frac{1.823 (0.01)^2}{1.89 \times 10^{-6}} = \underline{96.46 \text{ s}}$$

Results and/or Analysis of Results